Probability And Statistics

Assignment Set -I

(1) Define Vector space and Vector subspace.

Vector Space :- A vector space is a non-empty set V of objects called vectors on which are defined two operations called additions and multiplecation by scalors (real Numbers) subject to the axioms (or rules) listed below.-

- The axioms held for all vectors $\bar{u}, \bar{v}, \bar{w}$ in V and for all scalar c and d.

1. The sum of u and V denoted by $\bar{u}+\bar{v}$ is in ie $\bar{u}, \bar{v} \in \bar{V}$ (clocure property).
2. $\bar{u}+\bar{v} = \bar{v}+\bar{u}$ (commutative property)
3. $(\bar{u}+\bar{v})+(\bar{w}) = \bar{u}+(\bar{v}+\bar{w})$ (Associative property)
4. There is a zero vector 0 in V such that $\bar{u}+0 = 0+\bar{u} = \bar{u}$
5. For each $\bar{u}$ in V there is a vector $\bar{u} \in V \ni \bar{u}+(-\bar{u}) = 0$
6. The vector multiplication $c \in F, \bar{u} \in \bar{V} \Rightarrow c\bar{u} \in V$
7. $c(\bar{u}+v) = c\bar{u}+c\bar{v}$. (u, v are vectors). $c \in F, \bar{u} \in \bar{V}$ (c-scalar)
8. $(c+d)u = c\bar{u}+d\bar{u}$ (u-vector). $c, d \in F, \bar{u} \in \bar{V}$
9. $c(d\bar{u}) = (cd)\bar{u}$ $\quad c, d \in F, \bar{u} \in \bar{V}$
10. $1\bar{u} = \bar{u}$

Vector Subspace - A subset of a vector space V is called a subspace of V. if W is itself a vector space under the addition and scalar multiplication defined on V. some propertyes

(i) The zero vector of V is in H ie $\bar{0} \in H$

(ii) If $\bar{u}.\bar{v} \in H$ then $\bar{u}+\bar{v} \in H$

(iii) $\bar{u} \in H$ and c is any scalor then $c\bar{u} \in H$.

<u>Note</u>:- Let $\bar{u}, \bar{v}, \bar{w} \in H$

$\Rightarrow \bar{u}, \bar{v}, \bar{w} \in V \; (H \subseteq V)$

$(\bar{u}+\bar{v})+\bar{w} = \bar{u}+(\bar{v}+\bar{w})$

And $\bar{u}+\bar{v} = \bar{v}+\bar{u}$

we have $\bar{0} \in H$

$C\bar{u} \in H$

Let $c = -1$

$(-1)\bar{u} = -\bar{u} \in H$

H is subset of V

$\therefore \bar{u} \in H \Rightarrow \bar{u} \in V$

$\therefore C(\bar{u}+\bar{v}) = C\bar{u} + C\bar{v}$

$(c+d)\bar{u} = C\bar{u} + d\bar{u}$

$(cd)\bar{u} = c(d\bar{u})$

we have $C\bar{u} \quad H$

put $C = 1$

$\Rightarrow 1\bar{u} = \bar{u} \in H$

(2.) State and prove the addition theorem of probability.

If A and B are two events then the probability of occurance of A or B is given by

$$P(A) \text{ or } P(B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

when A and B are manually ~~expt~~ ~~e~~ exclusive events then

$$P(A \text{ or } B) = P(A) + P(B).$$

Proof:- Since events are sets from set theory, we have

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Dividing e.g. with $n(S)$; S - Simple space

$$n(A \cup B)/n(S) = n(A)/n(S) + n(B)/n(S) - n(A \cap B)/n(S)$$

Then by the adefination of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

(3.) Explain Sampling Techniques.

The Sampling techniques are used to examine the Selected sample from the population is known as Sampling Techniques. Sampling Technique is practical and its scope is vast. The whole data is analyzed with better supervision. It require less time and less cost. It gives reliable data the sampling Techniques ar as follows:

(i) Random Sampling
(ii) stratified Random Sampling
(iii) Systematic Random Sampling.

(i) Random Sampling – In this sampling, each item in the Population has an equal and likely possibility of getting selected in the sample. "Method of Chance Selection".

(ii) stratified Random Sampling – In this method, the Population is divided into subgroups to obtain a Simple Random Sample from each group and Complete the sampling process. the small groups are also ~~atleast~~ called strata.

(iii) Systematic Random Sampling – In this sampling method, the items are chosen from the destination Population by chosing the random selecting point and picking other methods after fixed Sample Period. This method is used for when a complete list of population is available.

Ex. - Consider there 1000 persons - from whom we have to chose 10 persons. for the study of any given Sample then we number them from 1 to 1000 and make them as 10 intervals and 1st we choose a person from the 1st pinteruel 2nd person is choosen from the 2nd interval and so on.

As we have numbered the persons all the persons are systematically choosen, This kind of method is used when the population is large.

(iv) Clustar Sample - A clustar sample randomly Selected group this method is usedful when the population quietly dispersed and Consists of many natural group such as factories, villages, etc.

(4) write about the testing hypothesis.

Testing of Hypothesis -

A Hypothesis is a statement about the population parameter. Hypothesis-testing is a procedure that helps us to ascertain the likelihood of hypothesized parameter being correct by making use of the sample statistics. The two hypotheses in a statistical test are Normally referred to as,

(i) Null Hypothesis
(ii) Alternative Hypothesis.

(i) Null Hypothesis - Null Hypothesis which is tested to be actually tested for acceptance or rejection is termed as Null Hypothesis. According to R A Fisher, "Null Hypothesis is the hypothesis which is tested for rejection under the assumption that it is true".

The Null Hypothesis is a very useful tool to test the significant of difference. In the process of statistical test, the Hypothesis is rejected or accepted based on the sample drawn from the mean of the population. This hypothesis reveals that the mean of the sample and the mean of the population under study do not show any difference.

A statistical hypothesis is a Null hypothesis if it is accepted. we should take consideration the followings while setting up the Null hypothesis:

(a) To test the significance of the difference between the values of the sample and the population or between two sample values; we set up the Null hypothesis that the difference is not significant. This is beacause the difference is due to sample fluctuations.

$$H_0: \mu = \bar{x}$$

where μ = population mean

$\bar{x}$ = sample mean

(b) To test any statement about the population, we set up the null hypothesis, that is the true.

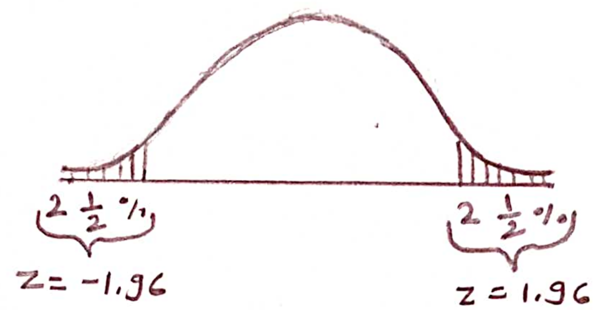
(ii) Alternative Hypothesis – "Any Hypothesis which is complimentary to the null hypothesis is called an alternative hypothesis.". Rejection of H_0 leads to the acceptance of Aletornative hypothesis which is denoted by H_1.

For example, if we want to test the null hypothesis for difference between population mean and sample mean ~~when~~ then we these hypothesis can be written as follows:

$H_0: \mu = \bar{x}$ (Null Hypothesis)

$H_1: \mu \neq \bar{x}$ (two-tailed alternative Hypothesis)

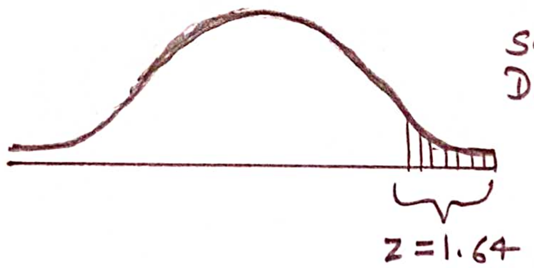
$H_1: \mu > \bar{x}$ or $H_1: \mu < \bar{x}$ (right tailed and left-tailed tests)



Two-tailed Diagram.

$H_0: \mu = X$

$H_1: \mu = X$



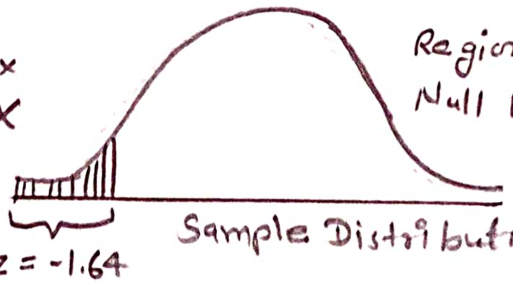
Sample Distribution

$C = X + z_x X\, S.E. =$ Critical value for upper time

$C = X - z\sigma \times S.E. =$ Critical value for left tail

$H_0: \mu = X$

$H_1: \mu = X$



Region of Acceptane of Null Hypothesis

Sample Distribution

The validity of H_0 and H_1 is then ascertained at a certain level of significant. The significance level stands for the confidence with which the experimentor rejects or retains the Null Hypo. Significance of level is customarily expressed as a percentage is in 1 in 20, when it is true.

Having setup the Null and the Alternative Hypotheses and the significance level, the next step is to construct & test criterion. This involves selecting the right probability distribution for the particular test.

5. Difference ~~between~~ Correlation of regression analysis

In many business situations we have to deal with two or more variables. specially, in the analysis and interpretation of data we have to take into account the relationship between demand and price, output and rainfall, volume of sales and expenditure on advertisement, etc. For study of such relationships the two important statistical methods used are correlation and regression.

These methods are also helpful in forecasting figures for the future. For example, a company planing next year's production may be interested in the forcast of sales for that year. if the marketing manager knows the sales having a relationship with advertising expediture and few other variables such as public expenditure, national income, etc.. he will be able to predict the value of ~~all~~ salse with these variable provided the value of all these variable is known. Similarly, a cost accountant can estimate the cost ~~and the price~~ of a product if there are estabillished relationship between the cost and the price of inputs such as labor, meterial, sales ~~prod~~ promotion expediture, etc. In statistics we find these relationships by the methods of correlation and regression.

CORRELATION	REGRESSION
(1) It determines the interconnection or a co-relationship between the variables.	It explains how an independent variable is ~~nu~~ numerically associated with the dependent variable.
(2) In correlation, both the independent and dependent values have no difference.	In regression, both the dependent and independent variables are different.
(3) The main objective of correlation is to find a quantitative or numerical value expressing the association between the values.	The main purpose is to calculate the values of a random variable based on the values of a fixed variable.
(4.) It stipulates the degree to which both variables can move together.	It specifies the effect of the change in the unit in the known variable (P) on the ~~eq~~ evaluated variable (~~q~~).(q).
(5.) It helps to constitute the connection between the two variables.	It helps in estimating a variable's value based on ~~anders~~ another given value.

Probability and Statistics

Assignment - II

(1.) Write about linear transformation and linearly independent sets?

Linear Transformations:-

Definition:- A transformation or (a function) T from R^n in R^n is said to be linear if

(1.) $T(\bar{u}+\bar{v}) = T(\bar{u}) + T(\bar{v})$ where $\bar{u}, \bar{v} \in R^n$.

(2.) $T(c\bar{u}) = cT(\bar{u}) \;\forall\, \bar{u} \in R^n \;\&\; c \in R$.

Ex:- Q if T is a linear tranformation then

(1.) $T(\bar{0}) = \bar{0}$

Proof:- Given T is a linear transformation therefore

$$T(\bar{u}+\bar{v}) = T(\bar{u}) + T(\bar{v}) \longrightarrow ①$$

$$T(c\bar{u}) = cT(\bar{u}) \longrightarrow ②$$

where c is scalar $\bar{u}, \bar{v}$ are vectors

To prove that $T(\bar{0}) = \bar{0}$

we have $T(c\bar{u}) = c.T(\bar{u})$

Put $c = 0$

$\Rightarrow T(0.\bar{u}) = 0.T(\bar{u})$

$\Rightarrow T(\bar{0}) = \bar{0}$

Linearly Independent sets :- An indexed set of vectors $\{\bar{v}_1, \bar{v}_2 \ldots\ldots \bar{v}_n\}$ is vector space V is said to be linearly independent if the vector equation $c_1\bar{v}_1 + c_2\bar{v}_2 + \ldots\ldots + c_n\bar{v}_n = \bar{0}$ has only the trivial solution $c_1 = 0, c_2 = 0, \ldots\ldots c_n = 0$

(1o) A set containing only one non-zero vector is always linearly independent.

$\bar{v}_1 \neq 0$

Let $c_1\bar{v}_1 = \bar{0}$

$\Rightarrow c_1 = 0$ or $\bar{v}_1 = 0$

Here $\bar{v}_1 \neq 0$

$$\therefore c_1 = 0$$

$\therefore \{\bar{v}_1\}$ is linearly independent.

(Q.) Define conditional probability and state and prove the multiplication theorem of Probability?

Conditional Probability:- A and B are two event in a sample space, then the conditional Probability of A/B (read as A given B) is defined as the Probability of A after the occurence of B and is given by

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Similarly $P(B/A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) \neq 0$

Multiplication Theorem:- The multiplication theorem of probability provides a way to find the probability of the intersection of two or more events.

Theorem Statement:- For Any two events A and B

(1.) Independent Event:

$$P(A \cap B) = P(A) \times P(B)$$

(2.) Dependent Event:

$$P(A \cap B) = P(A) \times P(B|A)$$

where $P(B|A)$ is the conditional probability of B given A.

Proof of Dependent Events:- for dependent event, the Probability of one event affects the Probability of the other. The multiplication rule for dependent event is:

$$P(A \cap B) = P(A) \times P(B|A)$$

Proof:- By the definition of conditional probability.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Rearranging this equation to solve for $P(A \cap B)$:

$$P(A \cap B) = P(B|A) \times P(A)$$

Thus, the multiplication rule for dependent event is prooved:

$$P(A \cap B) = P(A) \times P(B|A)$$

Proof for Independent Events:- Two events A and B are independent if the occurrence of one does not affect the occurrence of the other. This implies:

$$P(B|A) = P(B)$$

Hence, the multiplication rule for independent events becomes:

$$P(A \cap B) = P(A) \times P(B)$$

Proof:- by definition of independence

$$P(B|A) = P(B)$$

using the definition of conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

since $P(B|A) = P(B)$, we have:

$$P(B) = \frac{P(A \cap B)}{P(A)}$$

Multiplying both side by $P(A)$:

$$P(A) \times P(B) = P(A \cap B)$$

Thus, The multiplication rule for independent event is proved:

$$P(A \cap B) = P(A) \times P(B)$$

(3.) Describe discreate and continuous Distributions

Discreate Distribution:-

Definition:- Discreate distributions describe the probability of outcomes in a finite or countably infinite Set. The probabilitgies are assigned to specific values.

Key Features:- ① The sum of all probability is equal to 1.
(2.) Each individual outcome has a non-zero probability.
(3.) often represented using probability mass function (PMFs).

Examples:-

① Binomial Distribution :- Describes the number of successes in a fixed number of independent Bernoulli trials.
② Poisson Distribution :- Describes the number of events occuring in a fixed interval of time or space.
③ Geometric Distribution:- Describes the number of trials needed to get the first success in repeated independent Bernoulli trails.

Example formula (Binomial Distribution):-

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where n is the number of trials, k is the number of Successes, and p is the probability of sucess in each trails.

Continuous Distribution:-

Definition:- Continuous distributions describe the Probability of outcomes in a continuas range. Probabilityes are assigned to intervals rather than specific values.

Key features:-

① The total area under the probability density function (PDF) is equal 1.

② The probability of any specific value is zero; only intervals have non-zero probability.

③ often represented using probability density functions (PDFs).

Examples:-

① Normal Distribution:- Describes data that cluster around a mean (bell-shaped curve).

② Exponential Distribution:- Describes the time between events in a poisson process.

③ Uniform Distribution:- Describes a constant probability over a continuous interval.

Example formula (Normal Distribution):-

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean and σ is the standard deviation.

(4.) Explain point estimate, interval estimate, and confidence level.

Estimation of Population parameters :-

There are two types of estimates about a population parameters. -

(1) point estimates

(2.) interval estimate.

(1.) Point Estimate :- A single number used to estimate ~~an population parameter.~~ an unknown Population parameter.

Example: Sample mean ($\bar{x}$) estimate the population mean (μ).

(2.) Interval Estimate :- A range of values used to estimate a population Parameter.

Show the error by the extent of its range and the Probability of including the true population parameter.

Interval Estimate and Confidence Intervals :-

- Confidence Level : the probability that an interval estimate includes the population parameter.
- High Confidence level means greater certainty.
- Common confidence levels and corresponding Z-values:
- 90% $\pm$ 1.64
- 95% $\pm$ 1.96
- 98% $\pm$ 2.33
- 99% $\pm$ 2.58

Confidence Interval Estimate of the Mean :-

(1) Select confidence level:- Note the corresponding z-value.

(2) Compute Standard Error:-

(i) if population standard deviation (σ) is known:

$$\sigma_x = \frac{\sigma}{\sqrt{n}}$$

(ii) if σ is unknown, use sample standard deviation (S): $\hat{\sigma}_x = \frac{s}{\sqrt{n-1}}$

(3) Construct~~ion~~ Confidence Interval:-

(i) For known σ:- $\bar{x} \pm z\sigma_x$

(ii) For Unknown σ with large n:- $\bar{x} \pm z\hat{\sigma}_x$

(iii) For Unknown σ with small n:- $\bar{x} \pm t\hat{\sigma}_x$

Useful Confidence level and z-values :-

Confidence Level	Confidence Coefficient (z)
90%	1.64
95%	1.96
98%	2.33
99%	2.58

(5.) what is Annova ?

ANOVA (Analysis of Variance) :-

Definition :- ANOVA is a statistical technique used to compare the mean of three or more sample to determine if at least one of the sample means is significantly different from the others. it helps in understanding whether the observed variations among sample means are due to actual differences or random chance.

Types of ANOVA :-

(1.) One-Way ANOVA :- Used when comparing means of three or more independent groups based on one factor.

(2.) Two-Way ANOVA :- Used when comparing means based on two factors and it can also evaluate the interaction effect between the two factors.

Key Concepts :- (1.) Null Hypothesis (H_0) :- Assume that all group means are equal. $H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$

(2.) Alternative Hypothesis (H_a) :- Assume that at least one group mean is different.

(3.) F-Statistic :- Ratio of the variance between the group means to the variance within the groups. High F-value indicates that the group means are more spread out than expected by random chance.

(4.) P-Value :- Probability of observing the test results under the null hypothesis.